

# 薄壁压电陶瓷圆管机电参数的测定

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**摘要** 依据导出的切向极化和径向极化压电陶瓷薄壁圆管无负载电导纳方程, 给出由压电陶瓷圆管测量数据, 计算出机电参数的方法、公式和步骤。实验测知压电陶瓷薄壁圆管的尺寸、密度、低频电容、最大导纳频率与最小导纳频率以及其它若干频率上的电导纳等, 然后用给出的公式计算杨氏模量、泊松比、机电耦合系数、介电常数和压电常数等。运用这一方法, 对若干实际圆管样品作出了测量和计算。

## MEASUREMENT AND CALCULATION OF THE ELECTROMECHANICAL PARAMETERS OF THE PIEZOCERAMIC THIN CYLINDRICAL TUBE TRANSDUCERS

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**Abstract** On the basis of the electrical admittance equation for the unloaded tangentially and radially polarized piezoceramic thin circular cylindrical tube, the method and formulars, also the program for calculating the electromechanical parameters from the measured data are presented. If the size, density, capacitance at low frequency, frequencies at maximum and minimum admittances, and several other admittances with their corresponding frequencies are known from the measurement, then the Young's modulus, Poisson's ratio, electromechanical coupling coefficients, permittivity, and piezoelectric constants can be calculated with the formulars given here. Practical measurements and calculations for several samples have been carried out with this method.

### 一、引 言

压电材料机电参数传统地用简单振动模式测定<sup>[1,2]</sup>。铁电陶瓷圆管<sup>[3]</sup>等的机电参数除取决于陶瓷坯料外,还与极化有关。所以必须有一种对整管测定机电参数的方法,然后依据机电参数,用一定的理论关系<sup>[2]</sup>预知发射与接收特性。

## 二、无负载电导纳方程

在文献[2]中已指出,在横向各向同性假定下,对于无负载、薄壁圆管,作对称振动的状态方程,在切向极化情况下,可简化为

$$T_{rr} = 0 \quad (1)$$

$$T_{\theta\theta} = \frac{Y^E}{1 - \sigma^2} \{S_{\theta\theta} + \sigma S_{zz} - (\sigma + \tau)d_{31}E\} \quad (2)$$

$$T_{zz} = \frac{Y^E}{1 - \sigma^2} \{\sigma S_{\theta\theta} + S_{zz} - (1 + \sigma\tau)d_{31}E\} \quad (3)$$

$$D_\theta = d_{31}(\tau T_{\theta\theta} + T_{zz}) + \epsilon_{33}^T E \quad (4)$$

式中  $T_{ij}$  是应力,  $S_{ij}$  是应变,  $i, j = r, \theta, z$ ;  $Y^E$  是杨氏模量,  $\sigma = \sigma^E$  是泊松比,  $\tau = d_{33}/d_{31}$  是沿极化方向与垂直于极化方向的两个压电常数之比;  $D_i$  是电位移分量。命  $\tau = 1$ , 就得到径向极化下状态方程。

薄壁陶瓷圆管无负载时作对称振动,运动方程简化为<sup>[4]</sup>

$$\rho \ddot{u}_r = -T_{\theta\theta}/r, \quad \rho \ddot{u}_\theta = 0, \quad \rho \ddot{u}_z = \partial T_{zz}/\partial z \quad (5)$$

这对切向极化和径向极化都适用。因为是轴对称振动,所以  $\partial u_\theta/\partial \theta = 0$ , 从而  $S_{\theta\theta} = u_r/r$ 。对薄壁,近似有  $r = \bar{a} \approx (a + b)/2$ 。再假定沿轴向,电场分布均匀,即  $\partial E/\partial z = 0$ 。对于单频谐和振动,求出压电陶瓷圆管无负载电导纳方程

$$Y = -(1 - \sigma^2)\mu_{va}j\omega C^T k_{31}^2 / \mathcal{F}_a(\omega) + j\omega C_s \quad (6)$$

其中

$$\mu_{va} = \frac{1}{(1 - \sigma^2)^2} \left\{ \frac{\tan(K_a l/2)}{K_a l/2} \frac{[\sigma(\sigma + \tau) + \mathcal{F}_a(\omega)(1 + \sigma\tau)]^2}{[\mathcal{F}_a(\omega) + \sigma^2]} - (\sigma + \tau)^2 \right\} \quad (7)$$

此处  $\omega = 2\pi f$ ,  $f$  是频率,  $k_{31}^2 = Y^E d_{31}^2 / \epsilon_{33}^T = 4\pi Y^E d_{31}^2 / \xi_{33}^T$ , 并且有

$$\mathcal{F}_a(\omega) = (1 - \sigma^2)(\omega/\omega_r)^2 - 1, \quad \omega_r = c^E/\bar{a} \quad (8)$$

$$K_a^2 = (\omega/c^E)^2 \mathcal{F}_a(\omega) / [(\omega/\omega_r)^2 - 1] \quad (9)$$

$$C_s = C^T \left[ 1 - \frac{k_{31}^2}{1 - \sigma^2} (1 + 2\sigma\tau + \tau^2) \right] = C^T(1 - k_t^2) \quad (10)$$

$\rho$  是陶瓷的密度,  $c^E$  是陶瓷中声波的相速度,  $c^E = (Y^E/\rho)^{1/2}$ ;  $k_t$  是薄壁压电陶瓷圆管的机电耦合系数。可以求出在切向极化下低频自由电容为

$$C^T = NC_1^T = \frac{\xi_{33}^T N^2 l}{4\pi 2\pi} \ln\left(\frac{b}{a}\right) \quad (11)$$

式中  $N$  是沿圆管周向拼接的相同陶瓷片的数目,  $N$  片并联形成圆管总电容。若  $N$  以  $2\pi/\ln(b/a)$  代替,就得到径向极化下电容

$$C^T = \xi_{33}^T \frac{l}{2\ln(b/a)} \quad (12)$$

在公式中命  $\tau = 1$ , 就得出径向极化下的结果。圆管的电导纳曲线在两种极化下的区别将因比值  $l/2a$  的不同而有变化(图 1, 2)。例如当管长与直径相近, 即  $1 < (l/2a) < 10$ , 径向振动与长度振动耦合较强烈, 出现较多的耦合谐振峰; 对于切向极化圆管, 在径向共振频率

$f_r$  附近, 增加了尖峰的数目。泊松比  $\sigma$ , 横向机电耦合系数  $k_{31}$  和压电常数比值  $\tau$  对电导纳曲线的影响, 分别示于图 3, 图 4 和图 5 中。

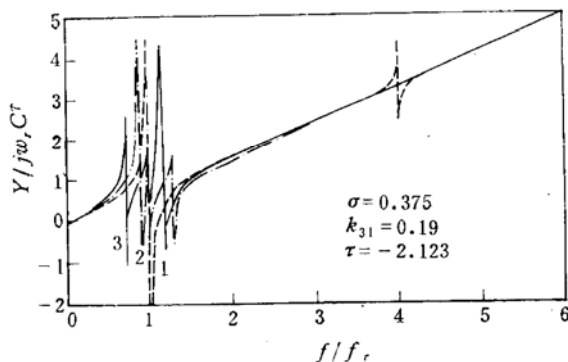


图 1 不同  $l/2a$  下切向极化薄壁压电陶瓷圆管的电导纳  
Admittance curves of the tangentially polarized piezoceramic thin circular cylindrical tube for various length to diameter ratio  $l/2a$   
1—0.42426; 2—1.5; 3—2.0

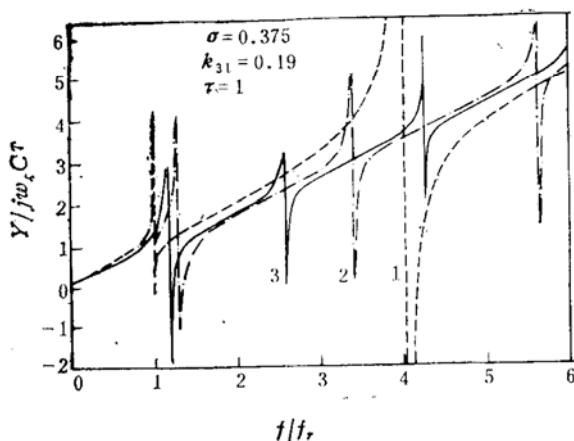


图 2 不同  $l/2a$  下径向极化薄壁压电陶瓷圆管的电导纳曲线  
Admittance curves of the radially polarized piezoceramic thin circular cylindrical tube for various length to diameter ratio  $l/2a$   
1—0.42426; 2—1.5; 3—2.0

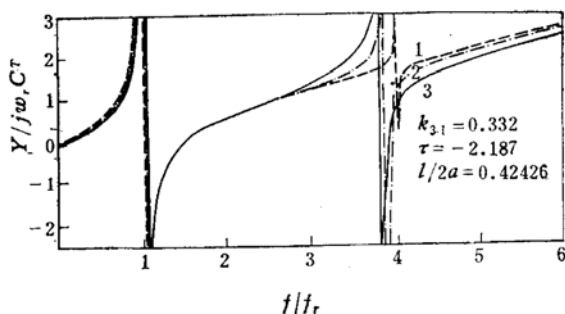


图 3 切向极化薄壁压电陶瓷圆管泊松比  $\sigma$  对电导纳响应的影响  
Admittance curves of the tangentially polarized piezoceramic thin circular cylindrical tube for various Poisson's ratio  $\sigma$   
1—0.375; 2—0.299; 3—0.215

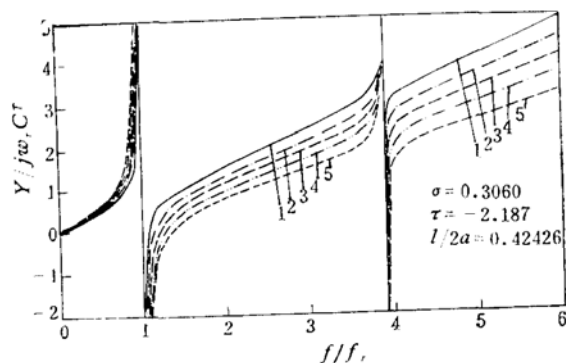


图 4 切向极化薄壁压电陶瓷圆管  $k_{31}$  对电导纳响应的影响  
Admittance curves of the tangentially polarized piezoceramic thin circular cylindrical tube for various electromechanical coupling coefficient  $k_{31}$ ,  
1—0.190; 2—0.228; 3—0.254; 4—0.280; 5—0.306

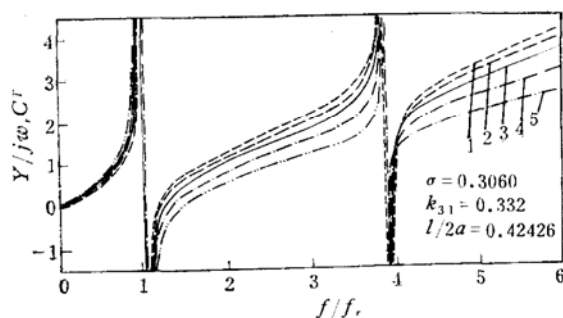


图 5 切向极化薄壁压电陶瓷圆管  $\tau$  对电导纳响应的影响  
Admittance curves of the tangentially polarized piezoceramic thin circular cylindrical tube for various piezoelectric constant ratio  $\tau$   
1—-1.564; 2—-1.680; 3—-1.800; 4—-2.000; 5—-2.187

### 三、机电参数计算法

由薄壁圆管频率方程

$$\omega_l^2[(\omega/\omega_r)^2 - 1] = \omega^2[(1 - \sigma^2)(\omega/\omega_r)^2 - 1] \quad (13)$$

可以得知

$$f_r = \left\{ \frac{1 + 1/(\pi \bar{a}/l)^2}{1 + (f_l/f_{II})^2} \right\}^{1/2} f_l \quad (14)$$

$$\sigma = \left\{ 1 - \frac{1 + (\pi \bar{a}/l)^2}{1 + (f_l/f_{II})^2} (f_r/f_{II})^2 \right\}^{1/2} \quad (15)$$

式中  $f_l \doteq f_{m1}$ ,  $f_{II} \doteq f_{m2}$ , 它们分别是在纯径向共振频率  $f_r$  和纯长度共振频率  $f_l$  附近的最大导纳频率。

在反共振频率  $f_a$  处导纳为零, 即  $Y(f_a) = 0$ 。对径向极化圆管, 有

$$k_{31}^2 = \left\{ \frac{2}{1-\sigma} + \frac{1+\sigma}{1-\sigma} \left/ \mathcal{F}_a(\omega_a) - \frac{\tan(K_a l/2)}{K_a l/2} \right. \right. \\ \left. \cdot \frac{[(1+\sigma)(f_a/f_r)^2 - 1]^2}{\mathcal{F}_a(\omega_a)[(f_a/f_r)^2 - 1]} \right\}^{-1} \quad (16)$$

算出  $k_{31}$  后, 再求压电常数  $d_{31}$ .

对切向极化圆管, 在  $f_a, f_1$  和  $f_2$  上测出电容, 依(6)式

$$C(f) = C^T \{ 1 + k_{31}^2 [\zeta(f) - (1 + 2\sigma\tau + \tau^2)/(1 - \sigma^2)] \} \quad (17)$$

命  $x = C(f_1)/C(f_2)$ ,  $x_a = C(f_a)/C^T$ ,  $x_1 = C(f_1)/C^T$ , 可知

$$\zeta_1 - x\zeta_2 - (x-1)\zeta_a = (x-1)x_a/k_{31}^2 \quad (18)$$

式中  $\zeta_1 = \zeta(f_1)$ ,  $\zeta_2 = \zeta(f_2)$ ,  $\zeta_a = \zeta(f_a)$ , 而

$$\zeta(f) = \tau^2 \bar{A}(f) + \tau \bar{B}(f) + \bar{C}(f) \quad (19)$$

并且有

$$\bar{A}_i = \bar{A}(f_i) = [\sigma^2(1+F)^2 G/H - 1/(1-\sigma^2)]/F \quad (20)$$

$$\bar{B}_i = \bar{B}(f_i) = 2\sigma[(1+F)(F+\sigma^2)G/H - 1/(1-\sigma^2)]/F \quad (21)$$

$$\bar{C}_i = \bar{C}(f_i) = [(F+\sigma^2)^2 G/H - \sigma^2/(1-\sigma^2)]/F \quad (22)$$

这里

$$F = \mathcal{F}_a(\omega), H = (1-\sigma^2)[\mathcal{F}_a(\omega) + \sigma^2], G = \tan(K_a l/2)/(K_a l/2) \quad (23)$$

于是得到

$$a\tau^2 + b\tau + c = (x-1)x_a/k_{31}^2 \quad (24)$$

其中

$$a = (x-1)\bar{A}_a + \bar{A}_1 - x\bar{A}_2 \quad (25)$$

$$b = (x-1)\bar{B}_a + \bar{B}_1 - x\bar{B}_2 \quad (26)$$

$$c = (x-1)\bar{C}_a + \bar{C}_1 - x\bar{C}_2 \quad (27)$$

但因为

$$x_1 - 1 = k_{31}^2 [\bar{A}_1 \tau^2 + \bar{B}_1 \tau + \bar{C}_1 - (1 + 2\sigma\tau + \tau^2)/(1 - \sigma^2)] \quad (28)$$

故进而有

$$\alpha\tau^2 + \beta\tau + \gamma = 0 \quad (29)$$

其中

$$\alpha = a - \frac{x_a(x-1)}{x_1-1} \left( \bar{A}_1 - \frac{1}{1-\sigma^2} \right) \quad (30)$$

$$\beta = b - \frac{x_a(x-1)}{x_1-1} \left( \bar{B}_1 - \frac{2\sigma}{1-\sigma^2} \right) \quad (31)$$

$$\gamma = c - \frac{x_a(x-1)}{x_1-1} \left( \bar{C}_1 - \frac{1}{1-\sigma^2} \right) \quad (32)$$

如果  $x_a = 0$ , 则  $\alpha = a$ ,  $\beta = b$ ,  $\gamma = c$ , 可由(24)式解出  $\tau$ ; 若  $x_a \neq 0$ , 则依(29)式解出  $\tau$ .  $f_1$  和  $f_2$  可取在低频段;  $f_1 < f_2$ , 则有  $C(f_1) < C(f_2)$ .

## 四、测量与计算

测量是用的共振与反共振频率及导纳圆图测量。低频自由电容  $C^T$  用精密电桥作辅助测量。用同一次测量各量代入公式计算；多次测量并计算的结果求平均。结果的校核是对同样的样品设法作简单振动模式的测量，对一些圆管换能器作了实际测量与计算，其结果列于表1—4。

表 1 PZT-4 径向极化圆管的实测值  
Measured quantities of PZT-4 radially-poled tubes

| 序 | 外 形  | $a$ (cm) | $t$ (cm) | $w$ (cm) | $l$ (cm) | $\rho$ (g/cm <sup>3</sup> ) | $f_{m1}$ (kHz) | $f_{m2}$ (kHz) | $f_{n1}$ (kHz) | $C^T$ (pF) |
|---|------|----------|----------|----------|----------|-----------------------------|----------------|----------------|----------------|------------|
| 1 | 圆 管  | 5.000    | 0.775    |          | 8.075    | 7.5                         | 9.895          | 22.437         | 10.008         | 28690      |
| 2 | 圆 管  | 1.000    | 0.125    |          | 0.900    | 7.5                         | 53.115         | 202.428        | 55.690         | 2641.6     |
| 3 | 轴向长条 |          | 0.125    | 0.15     | 0.900    | 7.5                         | 194.234        |                | 202.438        | 70.33      |

表 2 PZT-4 径向极化圆管的计算值  
Calculated quantities of PZT-4 radially-poled tubes

| 序 | $f_r$ (kHz) | $\sigma$ | $c^E$ (m/s) | $Y^E$ (10 <sup>10</sup> N/m <sup>2</sup> ) | $k_{31}$ | $\xi_{33}$         | $d_{31}$ (10 <sup>-12</sup> C/N) |
|---|-------------|----------|-------------|--------------------------------------------|----------|--------------------|----------------------------------|
| 1 | 9.9780      | 0.299    | 3109        | 7.2489                                     | 0.2179   | 921.56             | -73.056                          |
| 2 | 53.2535     | 0.2153   | 3549        | 9.5593                                     | 0.3063   | 622.27             | -73.485                          |
| 3 | 194.234     |          |             | 9.1676                                     | 0.3137   | 622.27<br>(736.49) | -76.850                          |

表 3 PZT 切向极化圆管的测量值  
Measured quantities of PZT tangentially-poled tubes

| 序 | 材 料   | 外 形   | $a$ (cm) | $t$ (cm) | $w$ (cm) | $l$ (cm) | $f_{m1}$ (kHz) | $f_{m2}$ (kHz) | $f_{n1}$ (kHz) | $f_{n2}$ (kHz) | $C^T$ (pF) | $\rho$ (g/cm <sup>3</sup> ) | $N$ |
|---|-------|-------|----------|----------|----------|----------|----------------|----------------|----------------|----------------|------------|-----------------------------|-----|
| 1 | PZT-5 | 圆 管   | 2.885    | 0.53     |          | 2.48     | 15.166         | 63.957         | 17.171         | 66.807         | 1687       | 7.35                        | 18  |
| 2 | PZT-4 | 圆 管   | 1.0      | 0.125    |          | 0.9      | 49.148         | 198.445        | 52.761         | 200.853        | 165        | 7.6                         | 8   |
| 3 | PZT-4 | 片拼圆管  | 1.0      | 0.125    |          | 0.9      | 44.334         | 196.448        | 51.273         |                | 306        | 7.6                         | 8   |
| 4 | PZT-4 | 弧形条   |          | 0.125    | 0.18     | 0.725    | 236.17         |                | 171.51         |                | 5.037      | 7.6                         |     |
| 5 | PZT-4 | 轴向直长条 |          | 0.125    | 0.15     | 0.9      | 191.03         |                | 195.197        |                |            | 7.6                         |     |

表 4 PZT 切向极化圆管的计算值  
Calculated quantities of PZT tangentially-poled tubes

| 序 | $l/2a$  | $a/t$  | $f_r(\text{kHz})$ | $f_a(\text{kHz})$ | $\sigma$ | $c^E$<br>( $10^9 \text{m/s}$ ) | $Y^E$<br>( $10^{10} \text{N/m}$ ) | $\tau$  | $(-k_{31})$ | $k_{33}$ | $\xi_{33}^T$ | $d_{31}^E$<br>( $10^{-12} \text{C/N}$ ) | $d_{33}^E$<br>( $10^{-12} \text{C/N}$ ) |
|---|---------|--------|-------------------|-------------------|----------|--------------------------------|-----------------------------------|---------|-------------|----------|--------------|-----------------------------------------|-----------------------------------------|
| 1 | 0.3951  | 5.9223 | 15.2163           | 15.930            | 0.3149   | 3000.9                         | 6.6192                            | -1.5635 | 0.1935      | 0.3027   | 884.6        | -66.516                                 | 104.00                                  |
| 2 | 0.42426 | 8.4853 | 49.416            | 52.761            | 0.3750   | 3293.3                         | 8.2426                            | -1.8969 | 0.1900      | 0.3601   | 1296.3       | -70.850                                 | 134.40                                  |
| 3 | 0.42426 | 8.4853 | 44.796            | 50.647            | 0.5218   | 2985.0                         | 6.6843                            | -2.0972 | 0.2295      | 0.4813   | 1182.4       | -90.763                                 | 190.35                                  |
| 4 |         |        |                   |                   |          | 3333.6                         | 8.4457                            |         |             | 0.5302   | 1182.4       |                                         | 195.32                                  |
| 5 |         |        |                   |                   |          | 3438.5                         | 8.9859                            |         | 0.2284      |          | 1182.4       | -77.907                                 |                                         |

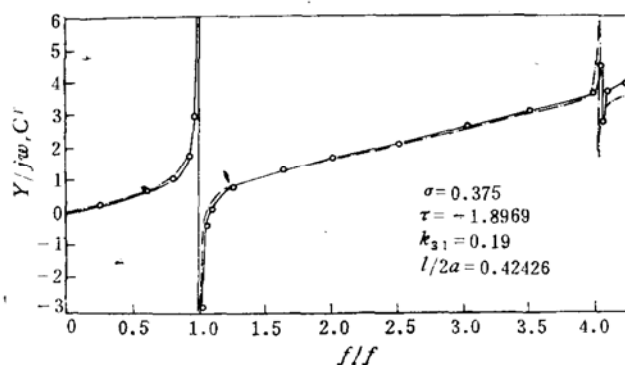


图 6 切向极化圆管导纳曲线理论与实验的比较

The theoretical and experimental admittance curves of the tangentially polarized piezoceramic thin circular cylindrical tube  
(— · — · — 理论曲线 theoretical, —○—○—实验曲线 experimental)

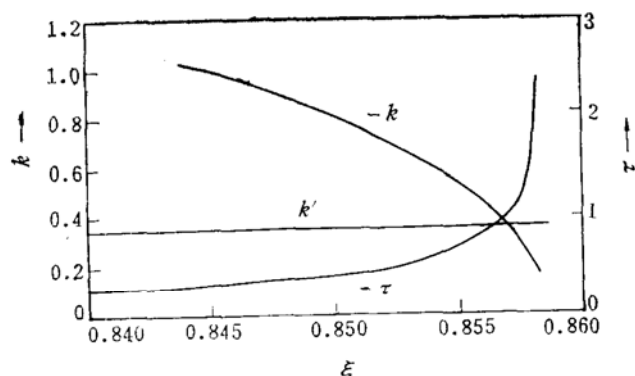


图 7 切向极化薄壁圆管求解  $k$ ,  $\tau$  和  $k' = k\tau$  等值随电容比值  $x = C(f_1)/C(f_2)$  的变化举例

Calculated  $k$ ,  $\tau$  and  $k' = k\tau$  vs capacitance ratio  $x = C(f_1)/C(f_2)$  for a tangentially polarized piezoceramic thin circular cylindrical tube

因边缘效应,长条片电容测量值偏高,而采用径向极化下整管电容测量值推算  $\xi_{33}^T$ , 则精度较高。这样,圆管与其上切下的长条片压电常数测量并计算值,相差在 5% 以内。

切向极化圆管样品 1 与 2 是在里外壁面沿周向等间隔涂细长轴向银面。样品 3 是在周向等间隔以切线为法线的截面上涂银,其导纳测量曲线与依据表中数据按 (6) 式画出的曲线,示于图 6; 二者符合较好。计算用编制程序在 TI-59 袖珍电子计算机上可迅速完成。

切向极化圆管自身电容小,杂散分布电容和电缆电容等影响低频电容比值  $x$  的准确性并给解算带来困难。我们在低于实测  $x$  值上假定其真值作计算;随着  $x$  下降,将可获得合理解。实算例如图 7 所示,可先求得乘积  $k\tau = k'$ 。用前面解得的  $\tau$  代入下式

$$k_{31} = \{(1 - z)/[\mu_{\nu a}/\mathcal{F}_a(\omega) + (1 + 2\sigma\tau + \tau^2)/(1 - \sigma^2)]\}^{1/2} \quad (33)$$

其中  $z = C(f)/C^T$ 。求出的  $k_{31}$  表为  $k''$ 。随着  $x$  变化,相应于其对应的  $\tau$  值的  $k$  与  $k''$  将相应变化。若  $z$  值正确,则  $k$  与  $k'$  的变化曲线,将有交点,在此点上  $k = k'' = k_{31}$ , 对应的  $\tau$  也随之确定并有  $k' = k_{33}$ 。为保证计算正确,测量导纳曲线要使电源稳压,仪器稳定可靠,恒温,无干扰,一次快速测完一条曲线,并对圆管结构工艺严格要求。所求得参数误差也可做到小于 5%,一般约在 20% 以内。

### 参 考 文 献

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